

endosperms, and in the prosenchymatous bast fibres, but is of much wider if not of universal occurrence. At any rate we were now in a position to get a clearer insight into such phenomena as the downward movement of a sensitive leaf upon stimulation, of the wonderful action of a germinating embryo on the endosperm cells, even on those which are most remote from it, of the action of a tendril towards its support, and of a series of phenomena in connexion with general cell mechanism, which were too numerous to mention, and could not be treated of in his present paper.

The paper is accompanied by forty figures, which illustrate the principal instances of protoplasmic continuity referred to in the text.

IV. "On the Dependence of Radiation on Temperature." By  
SIR WILLIAM SIEMENS, F.R.S., D.C.L., LL.D. Received  
April 25, 1883.

Sir Isaac Newton held that the radiation of heat from a hot body increased in arithmetical ratio with the difference of temperature between it and the surrounding bodies. This law forms a rough approximation to the truth over a very limited range of temperature. MM. Dulong and Petit carried out an elaborate experimental research on the rate of cooling of hot bodies by radiation, extending to somewhat higher temperatures, and deduced from their observations the empirical formula—

$$\text{Rate of cooling} = m(1.0077)^t(1.0077^{T-t}-1).$$

Here  $T$  is the temperature of the hot body in degrees Centigrade,  $t$  the temperature of the surrounding matter, and  $m$  is a constant depending on the nature of the radiating body. This formula agrees very fairly with experimental results for ordinary temperatures, but, like Newton's law, it has been shown that it cannot be applied for a wider range.

The anomalous results which Newton's law and the formula of MM. Dulong and Petit lead to, when applied to the cooling of bodies at a very high temperature, are well illustrated by the attempts at deducing therefrom the temperature of the solar photosphere. Waterston and Père Secchi (in his work entitled "Le Soleil"), following Newton's hypothesis, obtained  $10,000,000^\circ$  C. as the probable solar temperature, and Captain J. Ericsson, on the same hypothesis but assuming other constants, arrived at a temperature between  $2,000,000^\circ$  and  $4,000,000^\circ$  C. Strangely contrasting with these determinations are those of Pouillet in 1836, and Vicaire in 1872.

who, employing Dulong and Petit's empirical formula, deduce the values  $1461^{\circ}$  and  $1398^{\circ}$  C. for the solar temperature. Between these extreme estimates we have those of Dr. Spoerer,  $27,000^{\circ}$  C., of Zoellner,  $27,700^{\circ}$ , Professor James Dewar (1872),  $16,000^{\circ}$ , Rosetti (1878),  $9000^{\circ}$ , and Hirn (1882),  $20,000^{\circ}$ .

In my own investigations on this subject, by comparing the spectrum of the sun as regards the proportion of luminous rays with those of the electric arc and gas flames, I have arrived at the conclusion that the temperature of the photosphere does not exceed  $2800^{\circ}$  C., which is in close agreement with the limit assigned by M. Sainte-Claire Deville, deduced from the observations of Frankland and Lockyer on the hydrogen lines in the solar spectrum. Sir William Thomson, in a paper communicated to the Philosophical Society of Glasgow (1882), has compared the power of the sun's radiation per unit of surface with that of a Swan incandescent carbon filament, and has shown that it is about sixty-seven times greater; he concludes from these data that the estimate I had formed of the solar temperature, *i.e.*, nearly  $3000^{\circ}$  C., cannot be very far from the true value.

These diverse and indirect results have long impressed me with the need of further experimental investigation of the dependence of radiation on temperature; and it has occurred to me lately, that the difficulties with which Dulong and Petit had to contend in making their measurements by means of a mercurial thermometer, where the losses due to conduction and convection are very great, and exceedingly difficult to determine, might be avoided in adopting a method of conducting the experiment which forms the principal subject of my present communication.

It is well known that the measurement of electrical currents and resistance is susceptible of very great accuracy compared with all thermal measurements; hence my endeavour has been to estimate thermal effects entirely by electrical methods. In the Bakerian Lecture for 1871, which I had the honour of delivering before the Royal Society ("Proc. Roy. Soc.," vol. 19, p. 443), I showed that the resistance of a platinum wire can be expressed as a linear function of its temperature by an empirical formula, the constants of which must be determined for each individual wire; hence conversely, if resistance of a wire previously calibrated is measured, its temperature can be deduced. From theoretical considerations I showed that

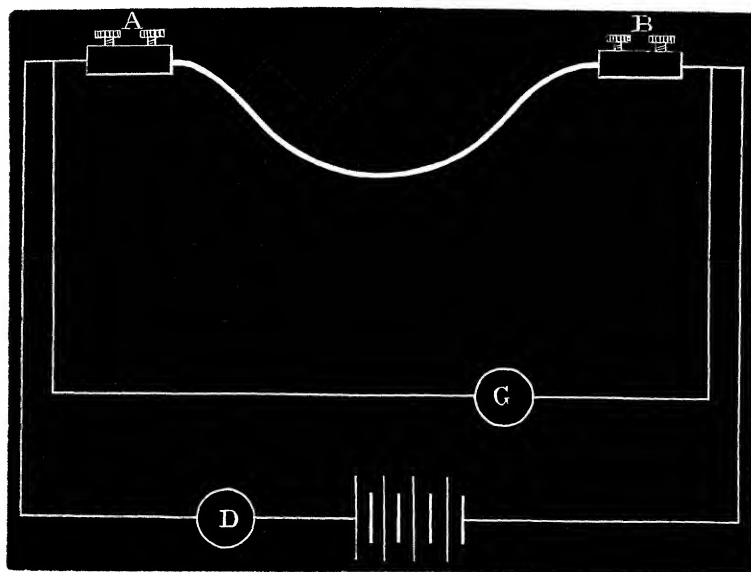
$$\frac{r}{r_0} = \alpha T^{\frac{1}{2}} + \beta T + \gamma$$

might be expected to represent the relation between the resistance and absolute temperature. This formula agreed closely with my own experimental results for platinum, copper, silver, iron, and aluminium wires ("Journal of the Society of Telegraph Engineers and Elec-

tricians," vol. i, p. 123, and vol. iii, p. 297), and has since been verified by Professor A. Weinhold in the case of platinum from  $100^{\circ}$  to  $1000^{\circ}$  C. ("Annalen der Physik und Chemie," 1873, p. 225).

The apparatus which I propose for determining the dependence of radiation on temperature consists of a platinum or other wire, 0.76 millim. in diameter, suspended between two binding screws, marked (A) and (B) on the diagram, carried on two suitable wooden stands. The binding screws are connected through an electro-dynamometer (D), for the purpose of measuring the current, to a secondary battery,

Diagram showing arrangement of experiment.



the number of cells in which can be varied. A high resistance galvanometer (G) is also inserted between the binding screws as a shunt to the platinum wire.

The electro-dynamometer is of the ordinary form, in which the current passes through a fixed coil, and a movable coil consisting of a single twist, hung by a torsion spring in a vertical plane at right angles to the plane of the fixed coil. The couple due to the current is balanced by the torsion of the spring, hence the angle of torsion is proportional to the square of the current. The current through the high resistance galvanometer being a measure of the difference of potential between the extremities of the platinum wire, the reading of the galvanometer, divided by the main current as determined by

the electro-dynamometer, is proportional to the resistance of the wire. Hence the constant of the instrument and the resistance of the galvanometer being known, the resistance of the platinum wire could be calculated, as the current was varied by altering the number of cells composing the battery.

The measurements were made in all cases when equilibrium had been established between the radiation and the energy of the current, as evinced by the constancy of the readings of the electro-dynamometer and galvanometer.

Having made a rough preliminary series of experiments to test the suitability of the method and apparatus, with satisfactory results, on April 17th I made a second series, the results of which are recorded in Table I. Column I gives the current in ampères passing through the wire; column II the difference of potential in volts between the terminals as deduced from the readings of the galvanometer; column III the rate at which the energy of the current was converted into radiant energy, represented by the product of the electromotive force and current, and therefore measured in volt-ampères or watts; column IV the resistance of the wire, being the ratio of the electromotive force to the current; column V the corresponding temperature of the wire in degrees Centigrade. Finally, column VI describes the condition of the wire as apparent to the eye.

Table I.

Length of wire 102 centims. Diameter 0·76 millim.

Temperature of room 65° F.

I.	II.	III.	IV.	V.	VI.
Ampères.	Volts.	Watts.	Ohms.	—	Just warm to touch.
2·91	1·192	3·468	·4096	—	
3·999	1·639	6·555	·4099	—	
5·738	2·831	16·24	·4933	100°	
8·943	5·662	50·64	·6331	282	
12·27	9·536	117·00	·7772	570	Chars wood.
16·66	16·39	273·0	·9838	881	Very dark red.
13·19	11·175	147·4	·8472	653	Red heat.
20·90	22·052	460·9	1·055	1075	Bright red.
23·73	26·82	636·4	1·130	1194	Very bright.

On April 18th, three further series of experiments were made, the results of which are set forth in a similar manner in Tables II, III, and IV.

Table II.

Length of wire 102 centims. Diameter 0·76 millim.

Current increasing.

Temperature of the room.	Ampères.	Volts.	Watts.	Ohms.	Corresponding temperature of wire.	
63·5° F.	2·565	·895	2·295	·3489	—	Just warm.
"	3·217	1·340	4·310	·4165	—	
"	6·36	3·204	20·377	·5037	120°	Hot.
"	8·511	5·146	43·798	·6046	250	
"	10·714	7·599	81·416	·7029	420	Chars cotton.
66·0	13·192	11·026	145·45	·8358	645	Discolouring.
"	13·698	11·927	163·38	·8707	690	Dark red.
"	15·595	14·602	227·72	·9363	816	Light red.
67·0	16·222	15·510	251·60	·9561	852	Bright red.
"	17·869	19·072	340·02	1·0698	960	Yellow.
"	25·094	29·80	747·86	1·1875	1260	White.

NOTE.—The temperatures corresponding to the very small currents are not given, as for very small deflections the electro-dynamometer readings could not be regarded as perfectly trustworthy.

Table III.

Length of wire 102 centims. Diameter 0·76 millim.

Current increasing.

Temp. of the room.	Ampères.	Volts.	Watts.	Ohms.	Corresponding temperature of wire.	
60° F.	2·744	·908	2·491	·3309	—	Just warm.
"	3·629	1·483	5·382	·4086	—	
"	6·79	3·278	22·258	·4827	125°	Hot.
"	8·995	5·364	48·251	·5963	270	Nearly chars cotton.
"	11·072	7·465	82·653	·6742	430	Chars cotton.
"	14·048	11·925	167·52	·8489	700	Dark red.
70°	16·247	15·496	251·76	·9538	855	Light red.
"	19·299	19·97	385·40	1·0348	1005	Bright red.
"	20·073	20·577	413·04	1·0251	1037	Very bright red.
"	22·948	25·643	588·45	1·1175	1164	Yellow.
"	23·634	26·25	620·40	1·1107	1185	Bright yellow.
"	25·171	28·31	712·59	1·1247	1240	White.
"	26·190	29·80	780·46	1·1379	1272	"

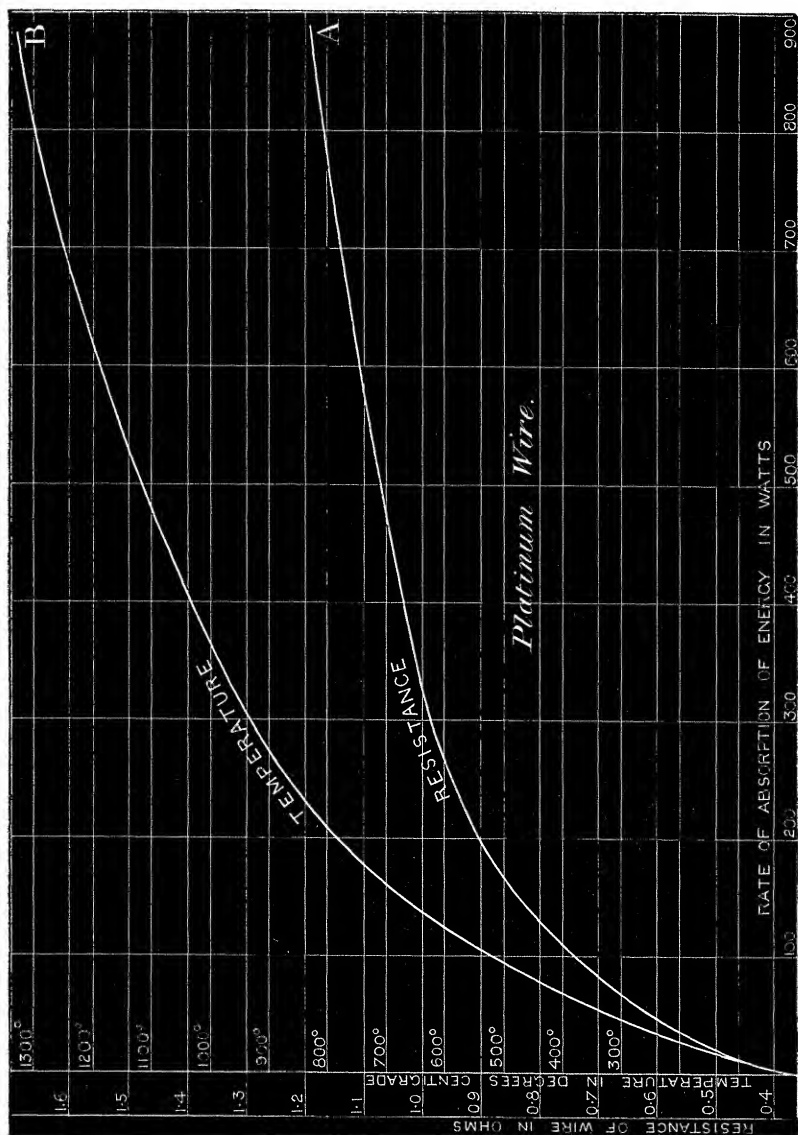


Table IV.

Wire the same as in III.

Current decreasing.

Tem- perature of the room.	Ampères.	Volts.	Watts.	Ohms.	Corre- sponding temperature of wire.
63° F.	25·101	28·31	710·61	1·1278	1240°
„	23·016	25·33	582·99	1·1005	1160
„	18·578	18·327	340·48	·9864	960
„	16·997	15·794	268·45	·9292	875
„	15·098	13·410	202·47	·8882	775
„	12·796	10·132	129·65	·7918	605
„	11·06	7·599	84·044	·6870	440
„	9·454	5·662	53·530	·5988	295
„	7·513	4·097	30·780	·5452	180
„	6·507	3·278	21·330	·5037	130
„	5·04	2·384	12·016	·4730	—
„	3·217	1·371	4·407	·4258	—
65°	26·856	31·29	840·33	1·1651	1290

The results given in the four tables are plotted out on the curve marked (A). The abscissæ give the rate at which the energy of the current is converted into heat, and the ordinates the corresponding resistance of the wire.

To determine the temperature of the wire corresponding to each resistance, another series of experiments was made, which are described hereafter. The values of  $\alpha$ ,  $\beta$ , and  $\gamma$  obtained were—

$$\left. \begin{aligned} \alpha &= 0\cdot0119 \\ \beta &= 0\cdot00112 \\ \gamma &= 0\cdot512 \end{aligned} \right\}$$

hence 
$$\frac{r_t}{r_0} = \cdot0119T^{\frac{1}{2}} + \cdot00112T + \cdot512;$$

where  $r_0$  is the resistance of the wire at the freezing point. By giving to  $T$  various values in this formula, a curve can be constructed showing the relation between the resistance and absolute temperature. Such a curve was drawn, and approximated for high temperatures to a straight line, as evidently must be the case from the form of the equation. By solving the equation for the maximum value of  $\frac{r_t}{r_0}$  observed, it was found that the temperature of the wire when bright red hot was about 1100° C. It is known that platinum wire melts at approximately 1800° C.

The curve of relation between the temperature of the wire and the

electrical energy absorbed can now be constructed. Taking the abscissæ of the curve proportional to the watts absorbed, and the ordinates proportional to the temperatures in degrees Centigrade, the dotted curve marked B represents the relation between the power and the temperature for the results given in the tables.

I have sought to express this relation by an empirical formula in order to carry the curve to still higher temperatures. The equation—

$$\text{Temperature} = A (\log x)^2 + B (\log x) + C,$$

where  $x$  represents watts, agrees with the experimental results. The constants A, B, C have the values,

$$A = -63.$$

$$B = 1177.$$

$$C = -1603.$$

Mr. McFarlane, in a paper communicated to the Royal Society on January 11th, 1872, has arrived at the equation—

$$\text{Rate of energy} = a + bt + ct^2,$$

where  $a, b, c$  are empirical constants and  $t$  is the difference of temperature, from his experiments made through a very limited range of temperature, viz., about  $60^\circ$  C. (*Proc. Roy. Soc.*, vol. 20, p. 90, 1872). Professor James Dewar, from experiments extending from a temperature of  $80^\circ$  to the boiling points of sulphur and mercury, also deduces a parabolic formula. (*Proceedings of the Royal Institution*, vol. 9, p. 266.)

Making use of the equation I have given, the rate of energy absorbed for a temperature of  $2780^\circ$  C., is 155,000 watts, or sixty-seven times the rate of absorption at a temperature of  $1670^\circ$  C. Since  $1670^\circ$  C. is not much below the temperature of an incandescent filament (reverting to Sir William Thomson's calculation for the ratio of the radiant power per unit of surface of the sun to that of the incandescent filament), the temperature of the sun comes out to be about  $2780^\circ$ ; which is in very close agreement with my former estimate based on other grounds. The effect of absorption between the sun and the earth would bring the two estimates into still closer agreement.

If we attempt to form a natural equation to the curve, it is apparent that it will consist of two terms—

(i.) The term due to radiation,

(ii.) The term depending on the convection and conduction of the air. The conduction of heat by the wire into the terminals may be neglected, as by taking a considerable length it becomes a small quantity of the second order. The first term I take to be proportional



to some power of the absolute temperature, the second may for the present be represented by  $mF(t)$ . Hence we have—

$$\text{Rate of conversion of energy} = AT^n + mF(t).$$

According to Prevost's theory of exchanges, the hot body is itself receiving radiant energy from the surrounding bodies; hence the radiant energy is more appropriately represented by  $A(T^n - t^n)$ , where  $t$  is the temperature of the surrounding bodies. Similarly it would appear probable that the conduction and convection will depend on the difference of temperature. Hence

$$\text{Rate of energy} = A(T^n - t^n) + mF(T - t).$$

The constants  $A$  and  $m$  will depend on the nature of the radiating body and on the surrounding medium.

Although for theoretical purposes it is important to eliminate the conduction and convection, yet in most cases a medium is present, and it has been shown by Mr. Crookes that, within limits, variations in pressure have only a very small effect on the amount of heat lost by conduction and convection.

I have not as yet been able to make any experiments on the determination of the term  $mF(T - t)$ , but it is my intention to make further investigations on this point. I am indebted to Professor Stokes for suggesting a method which appears to me likely to yield useful results. He proposes to construct a chimney of white paper, and to fix it over the wire through which the current is passing. The chimney will collect all the heated air ascending by convection, and by suitable means its temperature and the rate of flow can be measured, and hence the rate of loss of heat by convection estimated.

It might be supposed that conducting the experiment *in vacuo* would diminish the convection. According to the original researches of Dulong and Petit, the rate of cooling diminished in a geometrical progression, whose ratio was  $\frac{1}{1.366}$ , as the pressure diminished in a

second geometrical progression, of which the ratio was  $\frac{1}{2}$ . Mr.

Crookes, in a paper communicated to the Royal Society ("Proc. Roy. Soc.," 1880, vol. 31, p. 239) described some experiments on this point, and showed that a diminution of pressure from 760 millims. to 120 millims. had a very slight effect on the convection. From 120 to 5 millims. the effect was somewhat more marked. A reduction of pressure from 5 millims. to 2 millims., however, produced twice as much fall in the rate of cooling as the whole exhaustion from 760 millims. to 1 millim. Hence to eliminate the effect of convection a very high exhaustion must be obtained.

It still remains to describe the experiments by which the constants

$\alpha$ ,  $\beta$ ,  $\gamma$  of the empirical formula connecting the resistance of the wire with its absolute temperature were determined. The wire was enclosed in a glass tube, stopped at either end with a plug, through which the wire passed centrally. The tube was fixed in a metallic trough, with an aperture in its cover sufficiently large to admit a mercurial thermometer placed in contact with the tube. In the first instance, the trough was filled with melting ice, and the resistance of the wire measured by a Wheatstone bridge. The ice was then removed, and two Bunsen burners were placed below the trough, and the temperature gradually raised by increasing the pressure of the gas in the burners.

In this way a series of simultaneous observations were made of the temperature of the wire and its corresponding resistance up to  $100^{\circ}$  C. The results are given in the subjoined table. Care was taken at each reading that the thermometer had become stationary, and really represented the temperature of the wire. A second series of observations were taken as the wire cooled from  $100^{\circ}$  to zero; and the results are likewise given in the table.

Temperature rising.			Temperature falling.		
Temperature.	Resistance ohms.	$\frac{r_t}{r_0}$	Temperature.	Resistance ohms.	$\frac{r_t}{r_0}$
$0^{\circ}$ C.	5847	1.0000	$100^{\circ}$ C.	6827	1.1680
0	5837	—	97.7	6815	1.1660
0	5827	—	95.5	6798	1.1631
0	5827	—	90.0	6741	1.1533
66.3	6467	1.1064	78.5	6619	1.1324
66.6	6469	1.1068	76.6	6601	1.1294
67.2	6477	1.1081	62.5	6463	1.1057
68.5	6547	1.1201	48.3	6308	1.0792
70.2	6557	1.1218	46.6	6299	1.0777
72.2	6567	1.1235	32.2	6147	1.0517
81.6	6597	1.1286	31.6	6140	1.0505
85.0	6657	1.1389	21.6	6052	1.0354
86.1	6697	1.1458	0	5857	1.0000
93.2	6727	1.1509	0	5857	—
95.0	6747	1.1543			
98.8	6777	1.1594			
99.5	6817	1.1663			

For the reduction of the 26 equations obtained from these observations, the method of least squares was employed, giving

$$\alpha = 0.0119$$

$$\beta = 0.00112$$

$$\gamma = 0.512$$

The following are the results in substituting for the platinum a wire of platinum with 20 per cent. of iridium.

Diameter of wire  $\cdot 73$  to  $\cdot 75$  millim. Temperature of room  $59^{\circ}$  F.  
Length of wire 100 centims. Current increasing.

Ampères.	Volts.	Watts.	Ohms.	Corre- sponding temperature of wire.	Condition.
2·169	1·638	3·553	·7552	—	Just warm.
4·652	3·045	14·165	·6546	—	Warm.
6·858	6·815	46·742	·9936	442°	Hot.
10·17	11·745	119·48	1·1545	725	Chars cotton.
11·477	14·21	163·09	1·2381	873	Dark red.
12·932	16·67	215·58	1·2891	965	Red.
15·198	22·04	334·97	1·4502	1252	Light red.
17·807	29·00	516·40	1·6286	1587	Yellow.
20·791	36·25	753·67	1·7436	1787	White.
Current decreasing.					
16·762	24·65	413·19	1·4706	1289	
14·210	19·865	282·28	1·3980	1160	
11·828	14·935	176·65	1·2627	918	
10·62	12·76	135·51	1·2015	806	
8·40	8·845	74·299	1·0530	545	
5·487	4·93	27·051	·8985	279	
4·338	3·625	15·725	·8364	—	

A second series were taken with the same piece of wire, and the current increased until the wire broke.

Ampères.	Volts.	Watts.	Ohms.	Corre- sponding temperature of wire.	Condition.
2·743	1·907	5·23	·6952	—	Just warm.
7·062	7·005	49·47	·9919	439°	Hot.
10·492	12·66	132·86	1·2066	816	Chars cotton.
15·634	23·69	370·38	1·5153	1372	Light red.
19·324	33·53	647·93	1·7351	1771	White.
21·044	37·99	799·47	1·8053	1899	
22·414	41·72	935·10	1·8613	2001	
23·913	45·19	1080·60	1·8898	2053	Incandescent.
25·475	49·91	1271·50	1·9592	2185	
26·33	53·70	1413·90	2·0395	2325	Broke into several pieces immediately after reading.

The relation between the resistance and temperature is given in the following table.

Temperature rising.			Temperature falling.		
Temperature.	Resistance ohms.	$\frac{r_t}{r_0}$	Temperature.	Resistance ohms.	$\frac{r_t}{r_0}$
Melting ice, 0° C.	1·0072 1·0061 1·0061	1·0000	Boiling water ...	1·0924	1·0852
12·1° C. ....	1·0184	1·0117	13·8° C. ....	1·0198	1·0131
Boiling water....	1·0924	1·0852	Melting ice ....	1·0072	1·0000

The values for  $\alpha$ ,  $\beta$ ,  $\gamma$  deduced by the method of least squares are—

$$\alpha = \cdot 005$$

$$\beta = \cdot 000694$$

$$\gamma = -\cdot 7285$$

In conclusion I have pleasure in acknowledging the assistance I have received in conducting the experiments, and in the preparation of this paper, from Messrs. E. Lauckert and Edward Hopkinson, D.Sc.

The Society adjourned over Ascension Day to Thursday, May 10th.

Diagram showing arrangement of experiment.

